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LABORATORY STUDY OF WAVE ENERGY LOSSES BY BOTTOM FRICTION AND PERCOLATION

TECHNICAL MEMORANDUM NO. 31
BEACH EROSION BOARD
CORPS OF ENGINEERS

FEBRUARY 1953

FOREWORD

This paper is a report on the laboratory phase of a general investigation of the subject of wave energy losses by bottom friction and percolation. The field study is currently being conducted by the Agricultural and Mechanical College of Texas under a Beach Erosion Board contract.

This paper presents the results of a study made in the Research Division of the Beach Erosion Board in Washington, D.C. by Rudolph P. Savage, hydraulic engineer, with the assistance of John C. Fairchild, laboratory technician. The data were analyzed and the basic report was prepared by Mr. Savage. The report was put in final form for publication by Albert C. Rayner, Chief, Project Development Division. The opinions and conclusions expressed herein are not necessarily those of the Beach Erosion Board.

This paper is published under authority of Public Law 166, 79th Congress, approved July 31, 1945.

CONTENTS

DEFINITION OF SYMBOLS

A	Area of cross section of sand (cm^2)
B	Width of wave flume (inches or feet)
b	Length along wave crest (feet)
C	Correction factor for percolation equation
D_f	Average rate of energy dissipation by bottom friction (ft. lbs./sec./sq. ft.)
D_p	Average rate of energy dissipation by percolation (ft.lbs./sec./sq. ft.)
d	Water depth, from still water level to bottom (inches or feet)
GM_e	Median sand diameter (millimeters)
H	Wave height (inches or feet)
H_I	Initial Wave height (inches or feet)
H_r	Wave height remaining at 60 feet (inches or feet)
H_o/L_o	Wave steepness in deep water
Δh	Difference in hydraulic head (inches)
h_r	Ripple height, trough to crest (inches)
k	Transmission coefficient
K	Permeability coefficient of sand ($feet^2$)
K_f	Friction coefficient in the equation $\tau = K_f \rho V^2$ (cm./sec.)
L	Wave length (feet)
L_s	Length of sand column in sand permeability tests (inches)
M	Energy coefficient

$$M = \frac{\pi}{2 \tanh^2 \frac{2 \pi d}{L}}$$

m	Slope of sea bottom
p	Ripple pitch - length from crest to crest (inches)
P	Energy transmitted per unit time in crest length b (ft. lbs./sec.)
Q	Discharge (cm^3/sec.)
R	Semi-amplitude of horizontal displacement of water particles at sea bottom (inches or feet)
R_e	Reynolds number
T	Wave period (seconds)
t	Time (seconds)
V	Velocity (ft./sec.)
w	Unit weight of water (lbs./cu. ft.)
x,y	Coordinates with origin at sea bottom and beginning of sand beach
x	Distance of wave travel from beginning of test beach (feet)
Y	Vertical thickness of permeable bed (feet)
$-\alpha$	Modulus of decay for wave height loss on smooth side of tank ($ft.^{-1}$)
$-\alpha'$	Modulus of decay for wave height loss due to side friction ($ft.^{-1}$)
$-\beta$	Modulus of decay for wave height loss due to bottom friction or percolation ($ft.^{-1}$)
$-\theta$	Modulus of decay for both side friction and bottom friction or percolation ($ft.^{-1}$).
μ	Absolute viscosity of water (lbs.,sec.,/ sq. ft.)
ν	Kinematic viscosity of water (sq. ft./sec.)
ρ	Density of water
σ_ϕ	Phi standard deviation of sand
τ	Tractive force (lbs./sq. ft.)

LABORATORY STUDY OF WAVE ENERGY LOSSES BY BOTTOM FRICTION AND PERCOLATION

INTRODUCTION

1. Most existing theory concerning the movement of waves in shallow water, ($d/L < 0.5$), prior to breaking assumes that no energy is gained or lost by the wave in its travel. Although this assumption may be correct for certain conditions, and accurate enough for many applications of wave theory, it is not sufficiently correct nor adequate for others.

2. Theories developed by J. A. Putnam and J. W. Johnson(1,2) concerning the amount of energy lost by a wave, as it advances from deep water through shallow water to the breaking point, show that wave energy losses by bottom friction and percolation into a permeable bottom may approach 50 percent of the original energy under certain conditions encountered in nature on ocean shores. Figure 1 shows graphically the computed height and energy losses obtained for selected conditions, based on the theory by Putnam and Johnson. The conditions covered by the computations include beach slopes ranging from 1/20 to 1/300, wave periods from 6 to 18 seconds, and wave heights from 1 to 10 feet. A permeability factor of 100 darcys or 1.063×10^{-9} feet^2 (corresponding to a sand of 0.104 millimeter median diameter with a 35 percent porosity), and a friction parameter (K_f) of 0.01 were used in the computations involved in obtaining these figures. The friction factor is primarily dependent on the height and pitch of the sand ripples on the bottom(3). A reasonable average value of the factor appears to be 0.01. Figure 1 shows that as much as 25 percent reduction in wave height may occur as the wave moves from deep water to the breaking point for slopes of 1 on 300, and since the energy of a wave varies as the height squared, this represents a 44 percent reduction in wave energy. If this actually occurs, it can be seen that the amount of energy lost by waves travelling through shallow water may be an important factor, not only in nature, but also in model studies where testing is done with shallow water waves and an accurate determination of wave characteristics is not only desirable, but necessary. In nature such losses might be particularly important in areas such as the Gulf coast of the United States where gentle slopes extend far offshore.

3. The laboratory studies described herein were designed to test the theories on wave energy losses by bottom friction and percolation as presented by Putnam and Johnson insofar as possible in a small wave tank, and to obtain information which might lead to a reasonably accurate quantitative evaluation of these losses.

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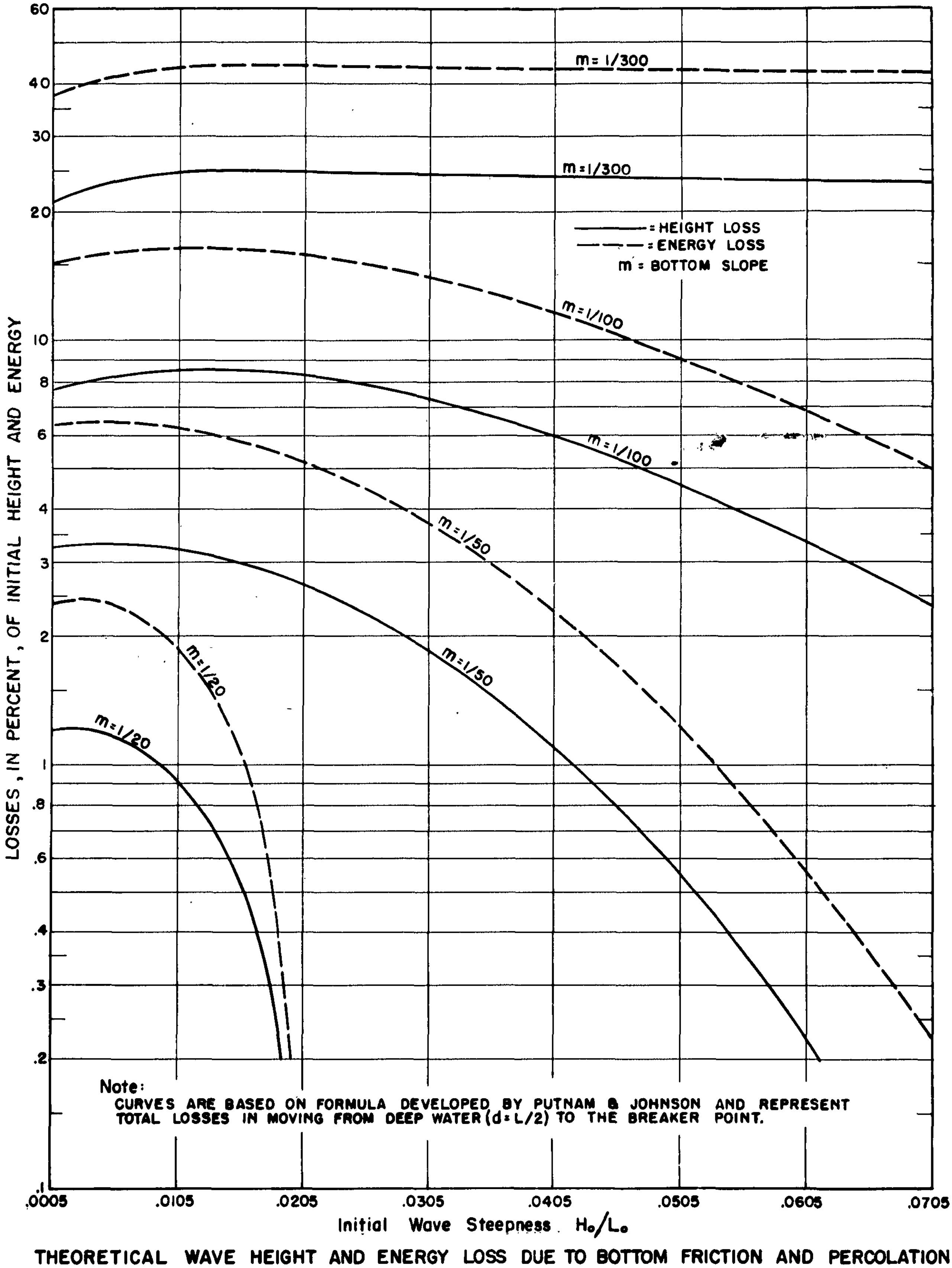
(1,2) Numbers in parenthesis refer to References in Appendix



THEORETICAL WAVE HEIGHT AND ENERGY LOSS DUE TO BOTTOM FRICTION AND PERCOLATION

Fig. 1

THEORETICAL METHODS

4. Wave Energy Loss by Bottom Friction. The theory for wave energy loss by bottom friction as developed by Putnam and Johnson(1), was based on the following basic assumptions:

a. the oscillatory motion at the bottom is sinusoidal and corresponds to the wave theory based on waves of small amplitude;

b. the friction coefficient does not vary with velocity and is approximately independent of water depth, this assumption being equivalent to choosing an average friction factor for the entire region over which the sea bottom affects the wave motion;

c. perpendicular flow at the bottom, resulting from percolation in the bottom material, is negligibly small; and

d. the sea bottom is a plane surface of constant slope upward to the breaker line.

5. Using an elementary equation of fluid friction, the equation for the tangential stress on a unit area at any instant ($\tau = K_f \rho V^2$), an expression is developed for the average rate of energy dissipation by bottom friction per unit area which is:

$$D_f = K_f \rho H^3 F(d,T) \tag{1}$$

where F(d,T) might be termed a "dissipation function" and is given by

$$F(d,T) = -\frac{4\pi^2}{3T^3}\left(\frac{1}{\sinh \frac{2\pi d}{L}}\right)^3 \tag{2}$$

If the origin for horizontal distance, x, is taken at the point where the depth equals one-half the wave length, the energy transmitted by the waves in unit time for a crest length of b_o at the station $x = 0$, where deep water conditions prevail, is given by

$$P_o = \frac{w\, b_o H_o^2 \sqrt{L_o}}{7.05}\left(1 - 4.935\,\frac{H_o^2}{L_o^2}\right) \tag{3}$$

For increasing values of x (toward shore) the bottom causes a change in wave height and wave length so that for a given wave period these two quantities will be functions of the depth, d. Assuming bottom friction to be negligible, the wave height may be computed from the following relation which is obtained by equating the energy of the wave motion corresponding to shallow water conditions to that corresponding to deep water conditions.

$$H^2 = \frac{16\, b_o P_o T}{w b L\left(1 - M \frac{H^2}{L^2}\right)} \tag{4}$$

where

$$M = \frac{4.935}{\tanh^2 \frac{2\pi d}{L}}$$

6. If the wave crests are not parallel to the shore line, a wave refraction diagram(4) must be prepared to obtain the trajectory of wave propagation and the variation of b and d along this trajectory. These values may then be plotted as a function of x for subsequent use in the calculations. For wave crests parallel to a straight beach no refraction occurs and b and b_o are everywhere equal.

7. In order to estimate the effect of bottom friction, equation 4 is used to compute a first approximation of H as a function of x. These values of H are then used with equation 1 to determine the energy dissipation as a function of x, resulting in a plot of the product $D_f b$ versus x. Graphical integration of this curve then yields the energy loss, ΔP, up to any desired value of x. Subtracting this energy loss from P_o then gives the energy available from which a new height is computed, using the relation:

$$\frac{H_f}{H} = \sqrt{\frac{P_o - \Delta P\left(1 - 4.935 \frac{H_o^2}{L_o^2}\right)}{P_o\left(1 - M \frac{H^2}{L^2}\right)}} \tag{5}$$

Strictly speaking, the H under the radical should be taken equal to H_f. However, this equation is small compared to the other terms and for convenience is taken equal to wave height neglecting friction. This process may be carried out until the desired degree of accuracy is obtained or until successive computed heights are equal.

8. The friction factor, K_f, used in equation 1 may be obtained from the results of an experiment conducted by Bagnold(3). The experiment dealt with the mean drag per unit area, τ, of an artificially rippled plate being moved with an oscillatory motion through water. Ranges of semi-amplitudes, R, from 5 to 30 centimeters and ripple pitches, p, of 10 and 20 centimeters were investigated. The pitch-height ratio (p/h_r) of the ripples was 6.7 to 1 and the ripple trough sections consisted of circular arcs meeting to form sharp crests at an angle of 120 degrees. The conclusions reached are summarized as follows:

a. If R/p is less than 1, K_f is a constant and is equal to 0.08

b. If R/p is greater than 1, $K_f = 0.072\,(R/p)^{-0.75}$

The extent to which K_f is dependent on the ratio p/h_r is not known, as a p/h_r ratio of 6.7 to 1 was the only one tested by Bagnold.

9. <u>Wave Energy Losses Due to Percolation in a Permeable Sea Bottom.</u> The theory for wave energy losses as a result of flow induced in a permeable sea bottom due to wave action was developed by Putnam(2) using the following assumptions:

a. the sea bottom at $y = 0$ is horizontal and the permeable material (sand) has a uniform permeability;

b. the sand at $y = 0$ does not move;

c. the water motion is two-dimensional;

d. viscous flow pervails in the sand;

e. the pressure at $y = 0$ varies sinusoidally with time and horizontal locations; and

f. the fluid is incompressible.

10. Using Darcy's law for the flow of a fluid through porous media and analyzing the problem as one in potential flow, an expression is developed for the average rate of energy dissipation (by percolation) per unit area, and is given as:

$$D_p = \frac{\pi \frac{K}{\gamma} \Phi_o^2 \rho}{L\left(1 + e^{\frac{-4\pi Y}{L}}\right)^2} \left[1 - e^{\frac{8\pi Y}{L}}\right] \tag{6}$$

where

$$\Phi_o = \frac{gH}{\cosh \frac{2\pi d}{L}}$$

This equation is rewritten in a dimensionless form and plotted in Figure 2* where it can be seen that when Y exceeds 0.3L, the depth of the permeable layer no longer has any appreciable effect on the energy dissipation and equation 7 may be written:

$$D_p = \frac{\pi K w^2 H^2}{L \mu \cosh^2 \frac{2\pi d}{L}} \tag{7}$$

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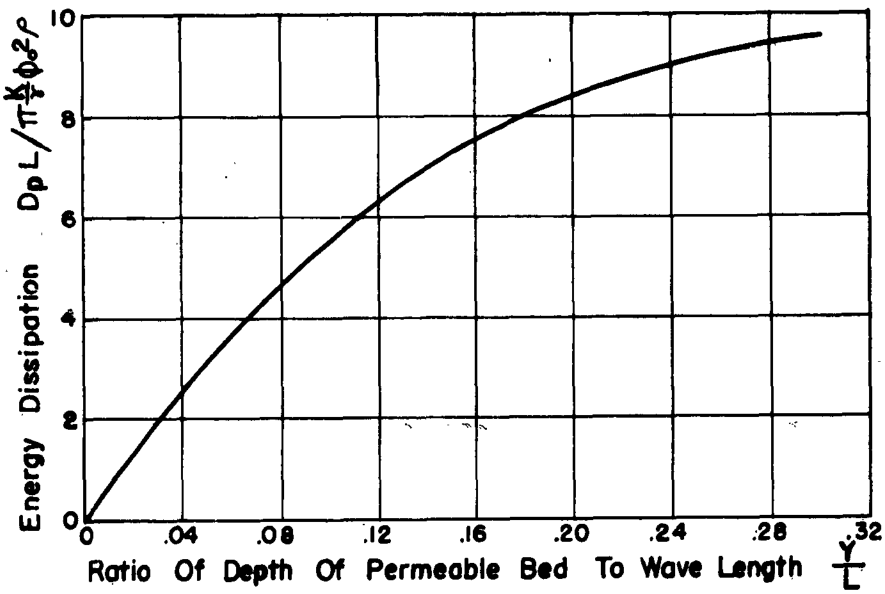
* It may be noticed that Figure 2 is the corrected curve for Figure 3 in the original Putnam and Johnson article in the Transactions, American Geophysical Union, which was in error. In this connection, it might also be noted that equation 18 of the original article is also slightly in error, the +2e in the second integral term of the equation correctly being -2e. These corrections have been checked with the authors.



EFFECT OF DEPTH OF PERMEABLE LAYER ON ENERGY DISSIPATION BY PERCOLATION
Fig. 2

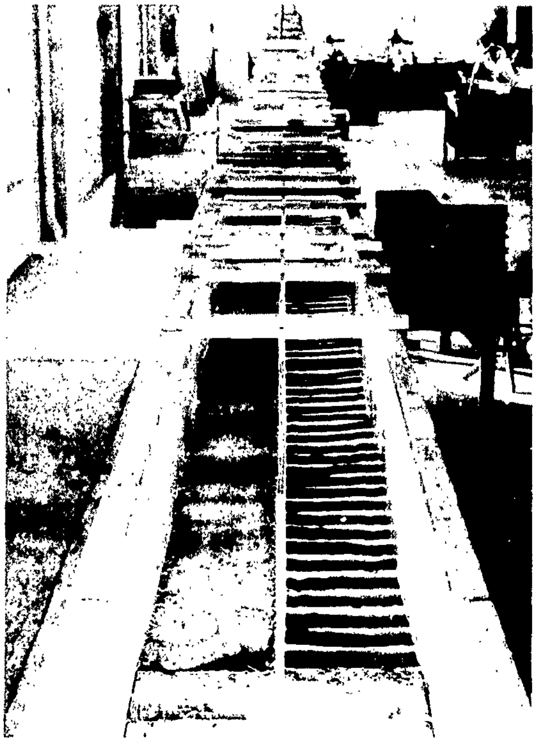
For smaller thicknesses of the permeable layer, the energy dissipation decreases exponentially. For a given wave period, both the wave length, L, and the wave height, H, depend on the depth, d, and may be determined from tables (5).

11. To estimate the order of magnitude of the reduction in wave height due to the dissipation of mechanical energy accompanying the viscous flow of water within the sand bottom, equation 7 may be used with a numerical integration method in a manner analogous to the procedure presented by Putnam and Johnson for reduction in wave height due to bottom friction and shown in paragraph 7.

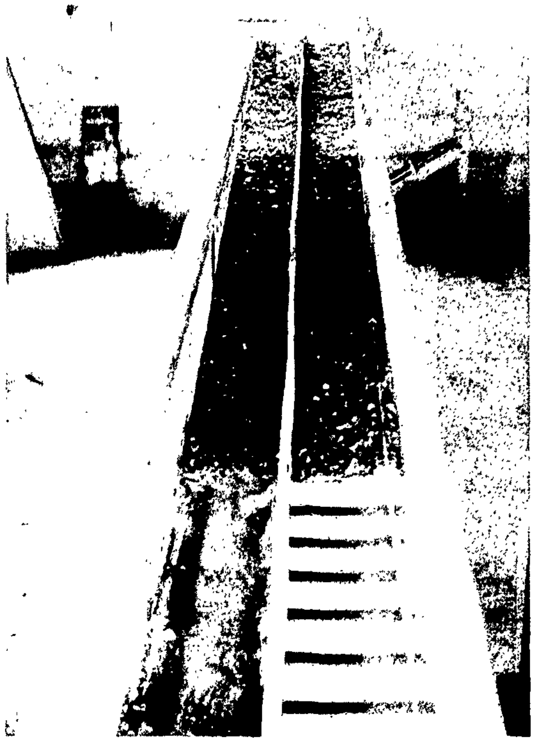
DESCRIPTION OF APPARATUS

12. Wave Flume. The experimental data were obtained in a wave flume of steel-plate construction, 96 feet long, 1.5 feet wide and 2 feet deep. For the purpose of these tests it was equipped with a plywood division or splitter wall beginning about 8 feet from the wave generator and dividing the tank in half longitudinally (Figure 3). Half the tank was used as a smooth side, or a side on which essentially no bottom friction or percolation effect would exist. On this side of the tank a plywood floor was installed one foot above the regular bottom of the tank. This plywood floor can be seen in Figure 3 on the left of the division wall and in the forepart of the picture as it continues toward the wave generator on both sides of the tank. The beach end of the tank containing the wave dissipator is shown in Figure 4. The wave dissipator consists of coarse crushed rock placed on a 1/20 slope facing the generator end of the tank.

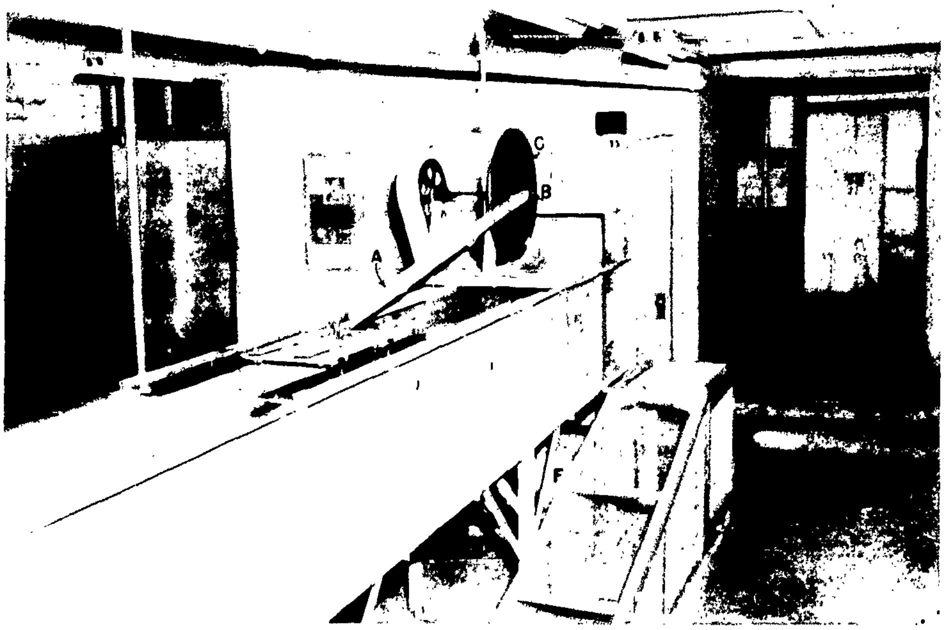
13. Wave Generator. The wave generator, shown in Figure 5, is of the vertical bulkhead, push-pull type. It is driven by a 1½ horse power



VIEW FROM GENERATOR END, SHOWING DIVISION WALL, SMOOTH AND RIPPLED SAND BOTTOMS.
Fig. 3



VIEW OF WAVE DISSIPATOR END
Fig. 4



WAVE GENERATOR AND DRIVE
Fig. 5

WAVE FLUME

electric motor through a Reeves vari-drive, (F). The wave periods obtainable with this drive range from 0.5 to 5 seconds. Strokes of the generator up to 11.5 inches may be obtained by moving the point of connection (B) of the connecting shaft (A) outward from the center of the flat steel wheel (C). Varying the stroke with any constant period gives wave heights as desired up to 12 inches, assuming adequate water depth for wave stability.

14. Wave Gages. The wave heights for runs 1 through 46 of this experiment were measured with an electrical step-resistance wave gage connected to a Brush recorder. This gage consists of resistors connected in series and encased in a non-conducting plastic in such a way that one plug of each of the resistors protrudes from the plastic, one above the other, forming a vertical path of resistance. A direct current voltage is impressed across this path of resistance, and as the water rises and falls along the length of the gage, contact with each protruding plug short circuits the resistor of that plug and a measure of water height on the gage is obtained by observing the resulting deflection of the pen on the Brush recorder. The gage is calibrated by incremental immersion in still water. This type of gage has two undesirable features. First, electrolytic deposits formed on the gage plugs and electrolytic gases which cling to the gage plugs block the flow of electric current between the gage and the water and cause a change in resistance of the gage with reference to the water making it necessary to calibrate the gage before and after each run. Second, the plugs of the resistors are spaced 0.2 inch apart, resulting in a possible error of as much as 0.4 inch in the measurement of a wave height. This latter error is compensated somewhat by the inertia of the recorder pen. Generally, a step-resistance gage will measure wave heights from 1 to 10 inches with a maximum error of about 10 percent.

15. In the remaining tests, the wave heights were measured with a combination hook and point gage. This gage was made by attaching two sliding verniers to a graduated brass rod. The bottom of this rod was hooked through 180 degrees, so that the end of the rod pointed upward when the main body of the rod was in a vertical position. The tip of the hook so formed was used to measure the elevation of the wave trough by raising or lowering the body of the rod with one of the verniers until the tip of the hook was at the same elevation as the wave trough. A pointed straight rod was attached to the other vernier and this rod was used to determine the elevation of the wave crest by raising or lowering the vernier until the tip of the rod just touched each wave crest as it passed. The distance between the tips of the straight rod and hooked rod is then the wave height and can be read from the verniers on the graduated brass rod. The most undesirable feature of this gage is that it is manually operated and is therefore subject to human error. However, if readings are carefully taken, wave heights from 1 to 12 inches can be measured with an error of 5 percent or less.

EXPERIMENTAL PROCEDURE

16. General. All tests were run in the 96-foot wave flume previously described. The side of the tank with the plywood floor was used as a control, since on this side essentially no bottom friction or percolation existed. The other side of the tank was used as a test side. Sands to be tested were placed in this side of the tank to a vertical thickness of one foot, which made the sand surface even with the plywood floor of the control side, both of which were level for the length of the flume. Since the loss in wave energy is proportional to the wave height squared, losses in energy could be determined by measuring the heights of the waves as they travelled from the beginning of the sand beach to the wave dissipator, a distance of about 70 feet. The interval along the walls of the tank between wave height measurements for most of the runs was 4 feet, but in some of the early runs, measurements were taken at 2-foot intervals. Measurements were made along both sides of the division wall so that energy losses could be compared. Tests for wave energy losses due to bottom friction were conducted using both artificial and natural ripples.

17. Friction Tests - Natural Ripples. One continuous friction test was made as follows: Sand with a median diameter of 0.216 millimeter was placed on the test side of the tank and smoothed to simulate the control side of the tank. Waves with a height of 2.5 inches and a period of 1.27 seconds were set in motion in 9 inches of water and allowed to run over the sand beach continuously for 24 hours or until natural ripple formation in the sand was completed. Beginning when the wave generator was started, measurements of the height of the waves as they travelled along the beach on both sides of the division wall were recorded at 1/2-hour intervals. These sets of heights were then tabulated to determine the effect on the wave energy loss of: (a) a smooth sand beach, (b) the formation and growth of the sand ripples, and (c) the sand ripples after they had reached full growth by wave action. The average height of the ripples formed in the test was 1/4 inch and the average ripple pitch was 1.25 inches.

18. Another continuous friction test was made starting from a smooth sand beach of the same sand. In this test, waves with a 1.27-second period were allowed to run over the beach until ripples were formed in the sand. The period of the waves was then changed to 1.00 second and these waves were allowed to run over the ripples left by the 1.27-second period waves. When no further change could be detected in the wave energy losses, the wave period was changed to 0.8 second and these waves were allowed to run over the ripples left by the 1.00-second waves. After the energy losses became constant for this condition, the wave period was changed back to 1.27-seconds and these waves were allowed to run over the ripples previously formed until no further change in energy losses was observed. The wave energy losses were determined by measuring the height of the waves over the 60-foot length of the sand beach with the hook and point gage.

19. Friction Tests - Artificial Ripples. Artificial ripples were formed in the sand by a screed board with sinusoidal notches. The ripples so formed were sprayed with sodium silicate to prevent their breaking up under wave action and to eliminate any possible percolation effect. These

artificial ripples had heights of one inch (trough to crest) and were formed in sand with a median diameter of 0.52 millimeter. Ripple pitches used were 1, 3 and 6 inches. Several later tests were made with artificial ripples sprayed with shellac rather than sodium silicate. These (shellac-covered) ripples were formed in sand which had a median diameter of 0.216 millimeter and were 0.31 inch in height and had a pitch of 1.25 inches. The wave periods and heights used varied between 0.8 and 1.27 seconds, and 0.31 and 3 inches. The water depth used in the friction tests was held constant at 9 inches. Height measurements similar to those obtained in the percolation tests were made to obtain values of energy loss due to bottom friction.

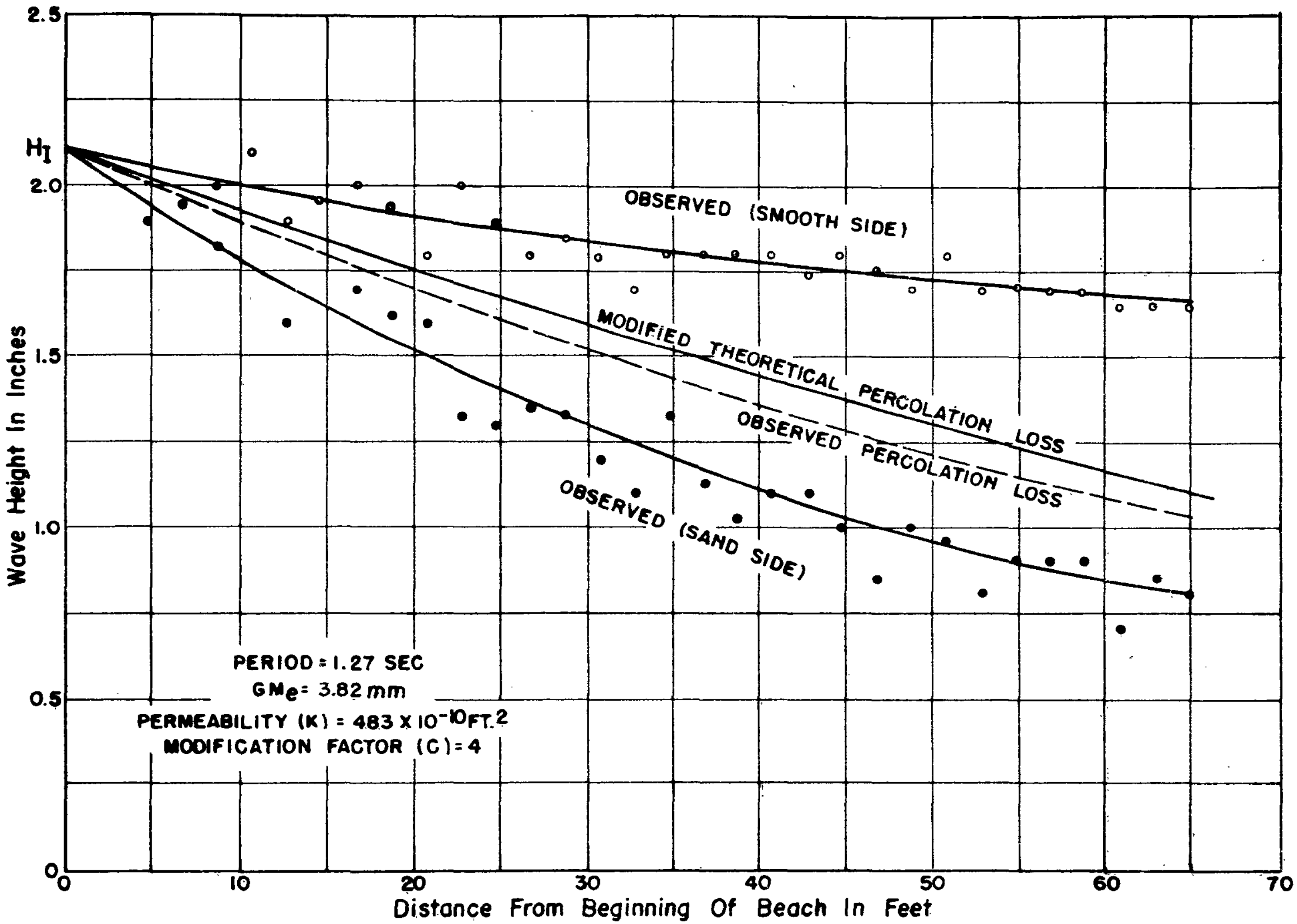
20. Percolation Tests. Tests were conducted for wave energy loss by percolation in a permeable bottom composed of sands with median diameters of 3.82, 1.94 and 0.52 millimeters. The sands had permeability coefficients of 483,231 and 6.02, respectively, determined by a permeameter, as described in the appendix. Water depths of 4, 6 and 9 inches were used. Wave heights used varied from approximately 1 inch to the maximum stable wave obtainable with any particular water depth and wave period, this being about 3.5 inches.

21. The percolation tests were hampered somewhat by the original condition that when the depth of the permeable layer, Y, is less than 0.3L, the energy dissipation decreases exponentially. In order to get maximum dissipation, either very short wave lengths or a large depth of sand had to be used. Since the sand, depth of water, and the part of the wave that is above still water level had to be contained in the 2-foot depth of the wave tank, a compromise had to be made and tests were made to determine the most reasonable water depths, wave periods, and sand thicknesses to be used. It was found that at very short periods, giving short wave lengths, only small waves (1 to 2 inches in height) could be generated, and these tended to oscillate crosswise in the tank, this oscillation becoming so pronounced at wave periods less than 0.8 second that wave periods shorter than this were unusable. With longer wave periods and the greater depths of water required to make the waves stable, the depth of water had to be so large that no room was left for the necessary depth of sand to keep Y/L more than 0.3. The results finally arrived at and used were:

Sand thickness - 12 inches
Water depths - 4, 6 and 9 inches
Wave periods - 1.27, 1.00 and 0.8 seconds
Wave lengths - 4" depth - 2.34, 3.05, and 3.60 feet
6" depth - 2.73, 3.60, and 4.30 feet
9" depth - 3.00, 4.16, and 5.04 feet

Under these conditions, the energy dissipation ranged from approximately 80 percent of the possible theoretical value for an unlimited thickness of sand with the 1.27- second period and 9-inch water depth, to 100 percent of the possible theoretical value with the 0.8-second wave period.

22. As the percolation tests with the 3.82-millimeter median diameter sand progressed and the 4 and 6-inch water depths were run, it was



LOSSES OF WAVE HEIGHT - RUN Nº 1

Fig. 6

found that an odd effect was present which made accurate determinations of wave heights difficult. This effect consisted of a separation of the wave as it was generated into what appeared to be the regular wave and a smaller wave that travelled faster than the regular wave. The crest of the smaller wave passed alternately through the trough and crest of the larger wave, increasing the larger wave height as it passed through the crest and decreasing the larger wave height as it passed through the trough. Because of this effect the smaller water depths were eliminated in the percolation tests with the 1.94-millimeter sand. Sand ripples on the bottom accentuated the effect and therefore the smaller depths were also eliminated in the friction tests.

EXPERIMENTAL RESULTS

23. General. The results obtained from both the friction and percolation tests assume their simplest form when plotted as wave height H versus distance from the beginning of the sand beach x, shown in Figure 6. From these a picture is obtained showing exactly what happens to the wave height as the wave travels over the sand and smooth beaches. Having these observed values, it must be determined how much of the wave height loss on the sand side is due to bottom friction or percolation and how much of the loss is due to side friction.

24. As shown by the change in wave height as the wave travels over the smooth beach, some of the height (and a corresponding amount of the energy) of the wave are dissipated by side and bottom friction, even on a smooth impermeable beach. It is necessary, therefore, to take these losses into consideration when deciding what portion of the height loss on the sand side is due to the sand bottom friction or percolation. At first, it would seem that the loss due to the sand bottom would be the difference in the wave heights on the smooth and sand sides at any particular distance, but this is not true, especially when the loss on the sand side is great. The difference between the wave height on the smooth and sand sides is not the true sand bottom loss because as the wave travels down the sand side, it is losing energy by side friction as well as by bottom friction or percolation. Consequently, the wave on the sand side is being reduced in height at a faster rate than the wave on the smooth side. Since the loss of wave energy by any means is proportional to the wave height, and the wave height at any particular distance on the sand side is smaller than the wave height on the smooth side, energy is not being dissipated due to side friction as rapidly on the sand side as it is on the smooth side and this effect increases with the distance of travel. Therefore, the total energy dissipated by side friction on the sand side is less than that on the smooth side, and the two values are not comparable.

25. Since the wave energy loss, and consequently the wave height loss, is a function of the wave height at any particular distance, it can be assumed that for a constant depth, such as is the case under consideration, the wave height loss follows the exponential law of decay as given by:

$$H_r = H_I e^{-\theta x}$$

where H_r is the height remaining at any distance x from the beginning of the sand beach, H_I is the initial wave height, and $-\theta$ is the modulus of decay. If this assumption is correct, an equation for the rate of change of wave height due to side friction with respect to distance can be written as:

$$\frac{dH}{dx} = -\alpha' H \tag{8}$$

where $-\alpha'$ is the modulus of decay for the side loss only. Similarly, an expression for the rate of change of H with respect to the distance can be written for the bottom friction or percolation losses on the sand side as:

$$\frac{dH}{dx} = -\beta H \tag{9}$$

where $-\beta$ is the modulus of decay for the bottom friction or percolation loss only. If these two effects are added, their results should be the height loss observed on the sand side and would be expressed as:

$$\frac{dH\ (\text{total})}{dx} = -(\beta + \alpha') H \tag{10}$$

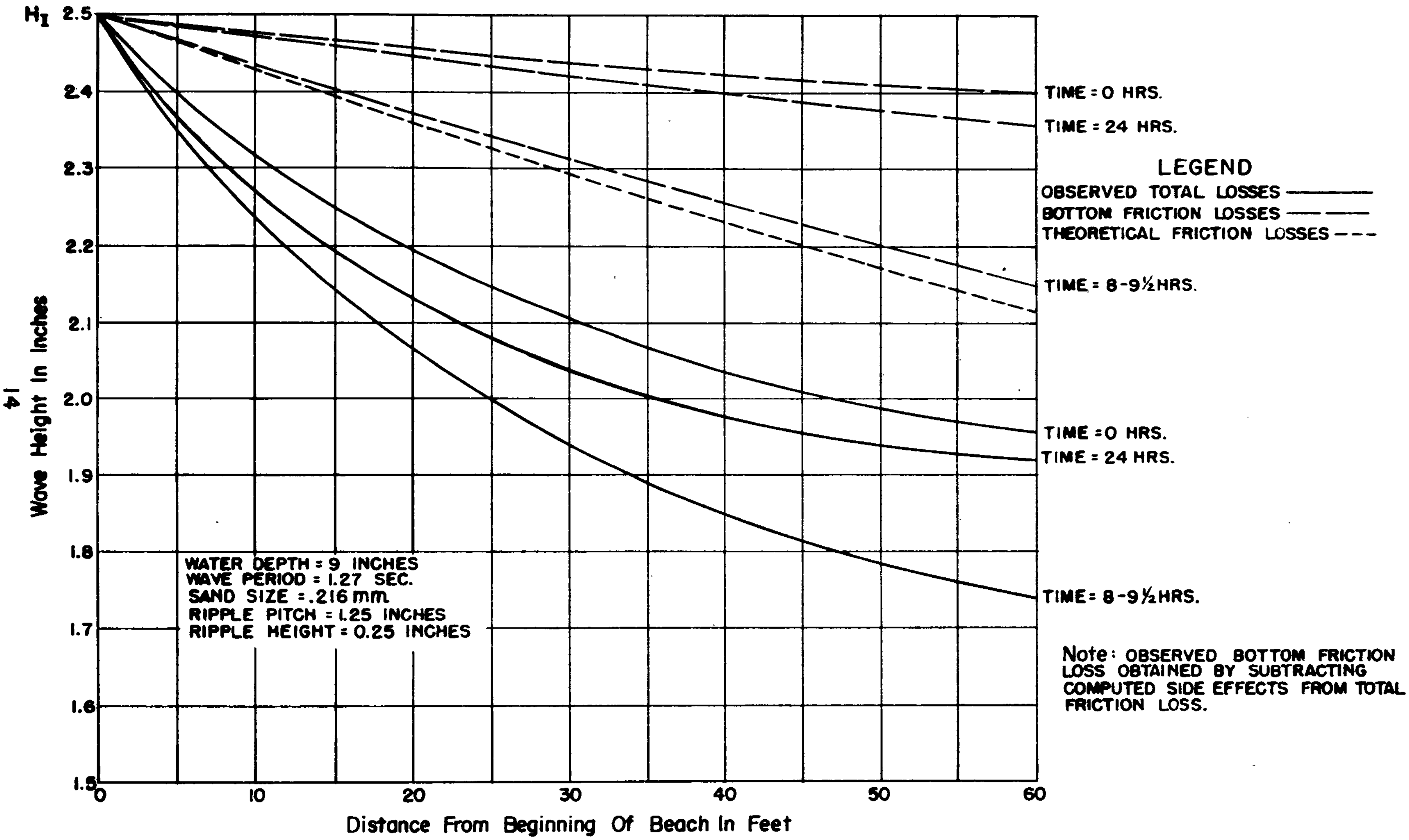
or

$$\frac{dH(\text{total})}{dx} = -\theta H \text{ where } -\theta = -(\beta + \alpha') \tag{11}$$

Since H_I, H_r, and x are known from measurement, θ can always be obtained for any particular set of conditions. Similarly, $-\alpha$ (the modulus of decay for the total losses on the smooth side) can be obtained for the same set of conditions from measurements on the smooth side.

26. In evaluating the wave energy losses due to percolation in the sand bottom (no ripples), it was necessary to eliminate the smooth side and bottom friction losses from the observed results on the sand side. This was done by subtracting the total smooth side friction losses from the observed percolation losses, and is tantamount to assuming that the friction on an unrippled sand bottom is the same as on the smooth board bottom. These losses are then compared with percolation losses as computed from the equations given by Putnam. In runs 12 through 29 of the percolation tests, the smooth board bottom was covered with a layer of sand one grain deep in order that the friction losses on the smooth side should be as nearly as possible equal to those on the sand side.

27. In order to eliminate as much experimental error as possible, the values of α for the smooth side were computed for all the friction and percolation runs. It was found that they were approximately constant for a given period and depth with the same bottom conditions, that is, a smooth board bottom or a sand covered impermeable bottom. The average α for each period, depth, and bottom condition was computed and used in all runs with the same conditions. The individual values of α did not vary from the average value by more than 10 percent and most values were within 5 percent of the average value. The results of these calculations are summarized in Table 1.



LOSSES OF WAVE HEIGHT-CONTINUOUS FRICTION TESTS WITH NATURAL RIPPLES & CONSTANT WAVE PERIOD

Fig. 7

TABLE 1

DECAY MODULUS α

Period seconds	Water depth inches	Average values of $\alpha \times 10^3$ Smooth Board Bottom	Board Bottom Covered with Coarse Sand
1.27	9	4.50	5.60
1.27	6	5.94	8.38
1.27	4	12.70	13.00
1.00	9	3.84	6.10
1.00	6	7.14	10.00
1.00	4	----	14.20
0.80	9	5.90	6.30
0.80	6	7.40	9.64
0.80	4	13.50	9.30

28. In evaluating the bottom friction losses, it was desired to substract from θ only that part of α that could be attributed to the side friction loss. The best estimate of the magnitude of this portion was found in an unpublished paper by Dr. G. H. Keulegan(6) concerning wave decay in a flume which gives:

$$\frac{\text{Mean bottom loss}}{\text{Mean loss for 2 sides}} = \frac{2\pi B}{L \sinh \frac{4\pi d}{L}}$$

Since the mean bottom loss plus the mean loss for both sides is equal to the total loss, an expression for the mean bottom loss in terms of the mean loss for both sides and the total loss can also be determined and is:

$$\text{Mean loss for 2 sides} = \frac{\text{total loss}}{1 + \frac{2\pi B}{L \sinh \frac{4\pi d}{L}}}$$

or, as it is used in this application,

$$\alpha' = \left[\frac{1}{1 + \frac{2\pi B}{L \sinh \frac{4\pi d}{L}}}\right]\alpha$$

where α' is the portion of α attributed to side friction alone. Values of $\frac{1}{1 + \frac{2\pi B}{L \sinh \frac{4\pi d}{L}}}$ vary only with period, provided the depth is

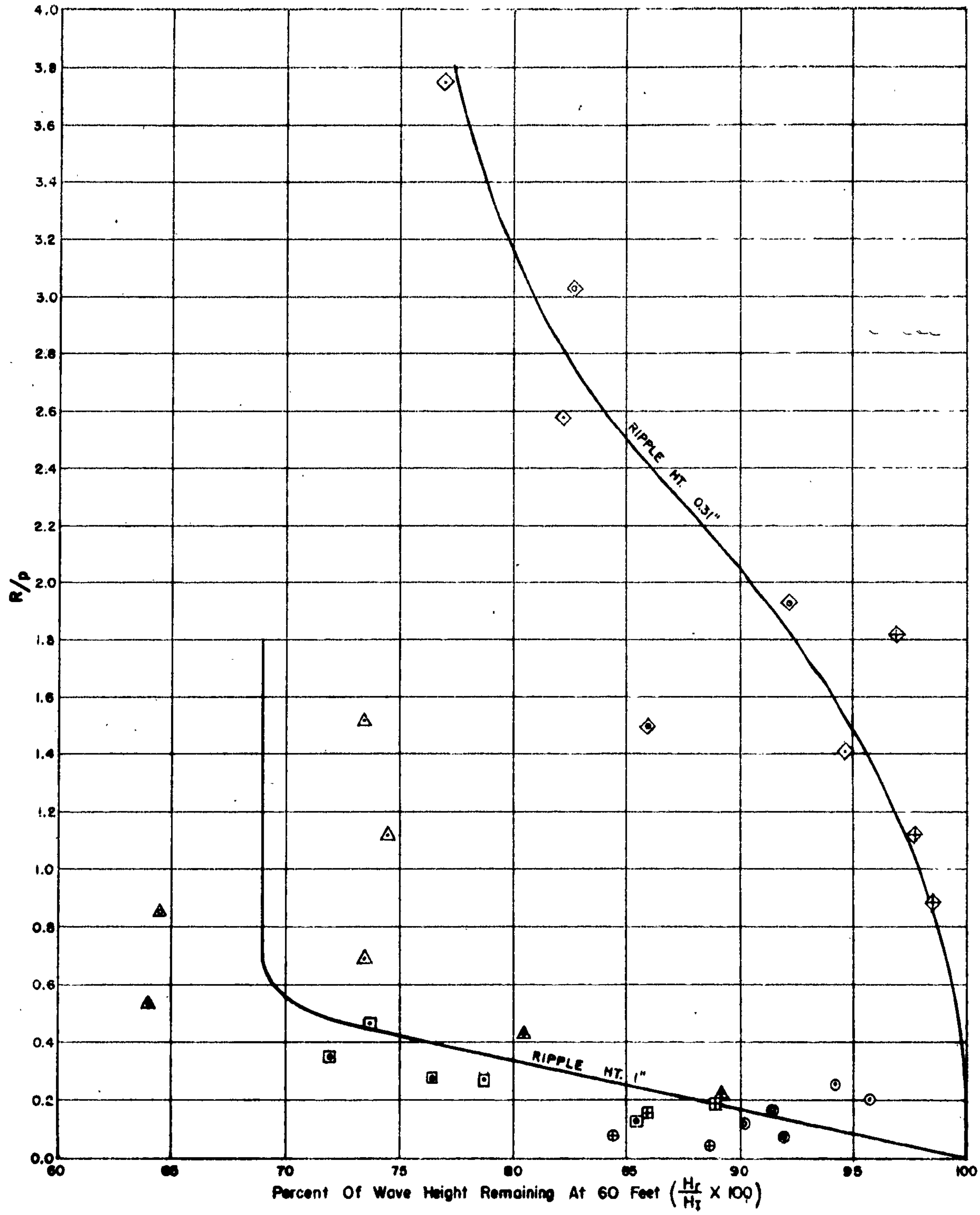
constant, as it is for all the friction tests. Therefore, values of the function were computed for all the wave periods used in the friction tests and were found to be:

Period seconds	$\dfrac{1}{1 + \dfrac{2\pi B}{L \sinh \dfrac{4\pi d}{L}}}$
1.27	0.755
1.00	0.809
0.80	0.847

Computing α' and subtracting it from θ yields β, the modulus of decay for the sand bottom friction only. β can then be used in conjunction with H_I and x to determine the height losses due to bottom sand friction effects only. The losses so computed were compared with friction losses as computed from equations given by Putnam and Johnson. The results are summarized in Tables 3 and 4.

29. Friction Tests - Natural Ripples. The results of the first continuous friction tests are shown graphically in Figure 7. It can be seen from the graph that the bottom friction loss at time equal 0 hours, or when the sand beach was still smooth, is relatively small, since at this time (if side effects are eliminated) 96 per cent of the original wave height remained at 60 feet. As the waves continued to run over the sand bottom, ripples began to form and the friction loss became greater, reaching a maximum at time equal to 8 to $9\frac{1}{2}$ hours. At this point 86 percent of the original wave height remained at 60 feet (if side effects were eliminated), and the ripple formation had progressed approximately one-half the distance from the beginning of the beach to the wave dissipator, or about 30 feet. As the ripple formation continued to advance toward the wave dissipator, the energy loss decreased until at time of 24 hours about 95 percent of the original wave height remained at 60 feet after taking into account the side effects, and the ripple formation had covered about 55 feet of the 60 feet of beach under consideration. After time of 24 hours, no further change occurred in energy loss or ripple formation.

30. The most significant fact revealed by this test is that the wave energy loss is greatest during the period when the greatest ripple formation is taking place. As the ripple formation continues, the wave energy loss decreases until the ripple formation is completed, at which time the wave energy loss is just slightly larger than it was on the smooth sand bottom. This indicates that it is not so much the ripples on the bottom that take energy out of the waves as it is the stage of the ripple formation and the compatibility of the wave train to that particular stage, or to the shape and size of the ripples at that particular time. This is logical in that Bagnold(3) found that sand ripples are a function of R, the semi-amplitude of the water particle motion at the bottom, up to certain limits which depend on the size of the sand. His findings can be summarized as follows:



RIPPLE HEIGHT = 1" Pitch = 1"	3"	6"	WAVE PERIOD (Sec.)	RIPPLE HEIGHT = 0.31" Pitch = 1.25"
◬	⊡	⊙	1.27	◇
▲	⊡	●	1.00	◈
▲	⊞	⊕	0.80	◇

FRICTION TESTS WITH ARTIFICIAL RIPPLES – EFFECT OF R/p ON FRICTION LOSSES

Fig. 8

starting from R = 0 with any sand, the ripple pitch increases as R is increased up to the "natural pitch" of the sand. After the "natural pitch" of the sand is reached, the ripple pitch does not increase no matter how much R is increased. The "natural pitch" of a sand is defined as a function of the size of the sand and varies approximately as the square root of the median diameter of the sand grains.

31. On the basis of these findings it appears that, up to the "natural pitch" of a sand, the size of the ripples in a particular sand is a function of the wave height, wave period, and water depth, since these are the parameters which determine R. If this is true, the ripples formed by any particular wave train of constant period and height will reach a state of equilibrium and little change will be observed in the ripples regardless of how long the wave train is allowed to run over them. The ripple pattern could then be said to be compatible to the wave train, and the energy losses as indicated by the continuous friction test would be in the range shown at time of 24 hours (Figure 7). Thus a "compatible" bottom is one which is hydraulically rather smooth in so far as the internal wave currents are considered. However, if the period and/or height of the wave train is changed, and the new wave train allowed to run over the ripple pattern formed by the old wave train, the ripple pattern is no longer compatible with the wave characteristics (hence hydraulically rough) and the energy lost by the waves should increase (as shown by the increase in energy loss at time of 8-9½ hours, Figure 7). The degree of severity of the increase in energy loss should depend upon the degree of incompatibility of the new wave train and the ripple pattern and the characteristics of the ripples themselves. From the foregoing it would seem that once the "natural pitch" of the sand had been reached and exceeded by R, the wave energy losses would always approach a maximum, since at this time the incompatibility of the wave characteristics and the ripple pattern would be most pronounced. This is somewhat substantiated by the tests made with artificially formed ripples, although the results are influenced by the fact that the ripples used in the tests were deformed compared to ripples observed under natural conditions. Figure 8 shows plots of R/p versus the percent of the wave height remaining at 60 feet for the tests with artificial ripples. From the graphs it can be seen that when R/p is small, the wave energy losses are relatively small, but as R/p increases the wave energy losses increase to a maximum and remain essentially constant for larger values of R/p in the range tested.

32. The theoretical friction loss as shown in Figure 7 was computed using the Putnam and Johnson equations and in this case agrees very well (within 2 percent) with the maximum bottom friction loss based on observations. Since no other tests were made with natural ripple formations, except one with varying wave periods (paragraph 33) it is not known just how well the theory would hold under other natural conditions.

33. The "compatibility" theory is also supported by the second continuous friction tests as shown in Table 2. After the 1.27-second period waves had run over the smooth sand beach for 7 hours, 91 percent of the wave height remained at 60 feet and even though the ripples had not reached

TABLE 2

WAVE HEIGHT LOSSES - FRICTION TEST WITH NATURAL RIPPLES
VARYING WAVE PERIOD

T (sec.)	Time (hours)	H_I (inches)	H_r (inches)	$\frac{H_r}{H_I} \times 100$ %
1.27	7	2.63	2.39	91
1.00	7	2.55	2.22	87
1.00	11	2.55	2.28	89
1.00	14	2.52	2.26	90
0.80	14	2.96	2.90	98
0.80	16	2.96	2.04	99
1.27	16	2.76	2.55	93
1.27	19	2.80	2.49	88
1.27	22	2.76	2.54	92
1.27	24	2.76	2.64	96
1.27	26	2.76	2.66	96

equilibrium, the wave period was changed to 1.00 second. Immediate measurements revealed that 87 percent of the wave height remained at 60 feet and measurements taken 4 hours later showed 89 percent of the wave height remaining at 60 feet. This indicated that the compatibility of the waves and the ripple pattern was increasing. Measurements after three hours had passed indicated that little change in the height losses was taking place, and the wave period was changed to 0.80 second. Measurements of these waves over a 2-hour period showed that there was little loss of height, and also little change in height loss, which can be explained by the fact that R was very small, resulting in very little turbulence and not much sand movement. The wave period was then changed to its original value or 1.27 seconds. Immediate measurement of the energy losses showed 93 percent of wave height remaining at 60 feet, and measurements taken 3 hours later showed 88 percent of the wave height remaining at 60 feet. Here it appears that in changing, the ripples went through a stage that caused more turbulence than the original ripple pattern created by the 1.00 and 0.80-second period waves. When this stage of ripple change had been completed, the energy losses began to decrease until the measurements were stopped about 7 hours later, at which time about 96 percent of the wave height remained at 60 feet.

34. The second continuous friction test, as described in the preceding paragraph, presents little that will help in determining a friction factor because the ripples could not be measured accurately without draining the wave tank. Refilling the wave tank would have destroyed the characteristics of the ripples and the test could not have been continued. However, this test does give some idea of the magnitude of the friction losses and the way these losses vary with the changing ripple pattern.

TABLE 3
WAVE HEIGHT AND ENERGY LOSSES
FRICTION TESTS WITH ARTIFICIAL RIPPLES

Note: H_{ro} indicates observed height at a point 60 feet from the beginning of the beach, adjusted to eliminate the effects of side friction, H_{rt} theoretical height.

Run No.	Period T (sec.)	Ripple Pitch p (inches)	H_I (inches)	R/p	H_{ro} (inches)	H_{rt} (inches)	$\frac{H_{rt} - H_{ro}}{H_{ro}} \times 100$ (%)	$\frac{H_{ro}}{H_I} \times 100$ (%)	Energy Loss (%)
Sand 0.52 mm. median diameter, Ripple height 1 inch									
58	1.27	6	2.24	0.202	2.14	1.83	-14	96	7
59	1.27	6	2.84	0.258	2.68	2.21	-18	94	11
60	1.00	6	1.25	0.075	1.15	1.13	- 2	92	15
61	1.00	6	2.74	0.164	2.50	2.30	- 8	91	17
62	1.00	6	1.95	0.116	1.76	1.70	- 3	90	18
63	0.80	6	2.20	0.080	1.86	2.01	18	84	29
64	0.80	6	1.30	0.047	1.15	1.22	6	88	22
49	1.27	3	1.55	0.278	1.22	1.33	9	79	38
50	1.27	3	2.60	0.467	1.92	2.02	6	74	45
51	1.00	3	2.30	0.276	1.76	1.95	10	77	42
52	1.00	3	1.10	0.132	0.94	1.01	7	85	27
53	1.00	3	3.00	0.350	2.16	2.40	11	72	48
55	0.80	3	2.70	0.197	2.40	2.40	0	89	21
56	0.80	3	2.20	0.161	1.89	2.01	6	86	26
65	1.27	1	2.84	1.530	2.08	2.37	14	73	46
66	1.27	1	2.08	1.120	1.55	1.71	10	74	44
67	1.27	1	1.28	0.690	0.94	1.13	21	73	46
68	1.00	1	2.37	0.850	1.53	1.99	30	64	58
69	1.00	1	1.50	0.538	0.96	1.34	40	64	59
70	0.80	1	2.00	0.438	1.61	1.84	14	80	35
71	0.80	1	1.03	0.226	0.92	0.98	7	89	21
Sand 0.216 mm. median diameter, Ripple height 0.31 inch									
4a	0.80	1.25	1.23	0.89	1.21	1.15	-5	98	3
5a	0.80	1.25	2.52	1.83	2.44	2.32	-5	97	6
6a	0.80	1.25	1.60	1.12	1.57	1.50	-4	98	4
7a	1.00	1.25	1.33	1.50	1.16	1.20	3	87	26
8a	1.00	1.25	1.71	1.93	1.57	1.54	-2	92	15
9a	1.00	1.25	2.68	3.03	2.21	2.25	2	83	32
10a	1.27	1.25	2.58	3.75	1.99	2.16	9	77	41
11a	1.27	1.25	1.78	2.58	1.46	1.46	0	82	32
12a	1.27	1.25	0.97	1.40	0.92	0.86	-7	95	10

35. Friction Tests - Artificial Ripples. The observed heights remaining at 60 feet adjusted to eliminate effects of side friction are given in Table 3 together with theoretical heights based on Putnam and Johnson's equations. As can be seen, the theoretical heights remaining at 60 feet do not agree well in most cases with the observed heights, the disagreement varying with wave period, wave height and ripple pitch. This disagreement with different ripple pitches can be explained by the fact that the friction factor K_f, used in the theoretical equations does not take into consideration the varying ripple pitch-height ratio, since the theory for the determination of K_f as set forth by Bagnold considers only the ripple pitch-height ratio of 6.7 to 1. The disagreement with different wave periods and heights can be explained only in terms of the theoretical equations and indicates that the equations are unsatisfactory for the prediction of bottom friction losses under the conditions tested.

36. The results shown for Runs 4a to 12a in Table 3 were obtained using ripples with dimensions closer to those forming naturally in the first continuous friction test. Here, the theoretical heights as predicted from the equations of Putnam and Johnson and the adjusted heights based on observations agree within the limits of experimental error.

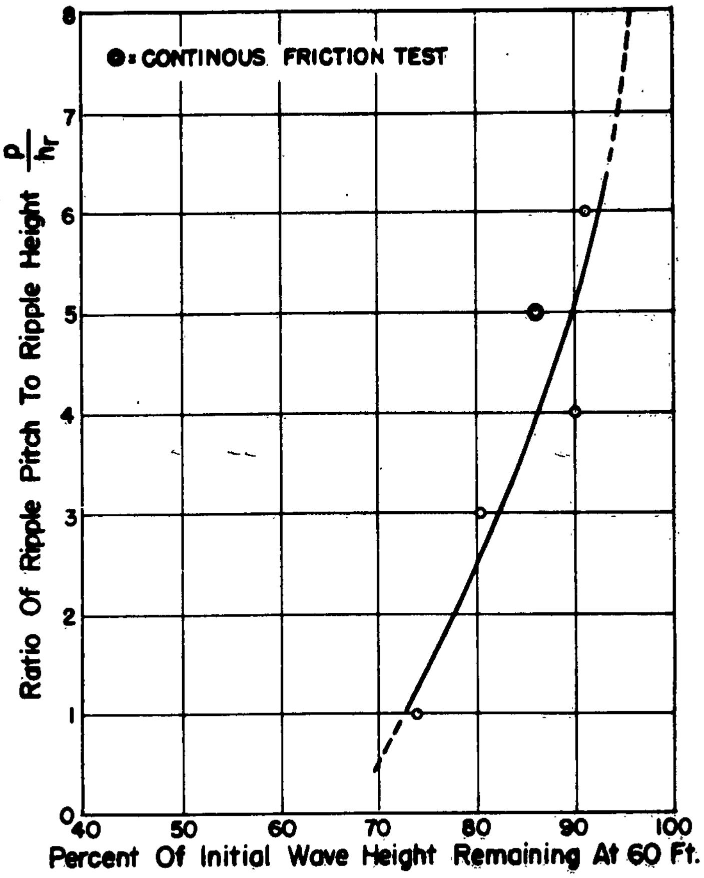
37. In general, the test results, as shown in Table 3, indicate that friction losses increase with decreasing ripple pitch-height ratio. A rough estimate of the effect of the ripple pitch-height ratio on the energy losses can be obtained by taking the average of all the observed values of the percentage of the wave height remaining at 60 feet for each ripple pitch plotted against the ripple pitch-ratio, as shown in Figure 9. The upper limit of the curve, which approaches 96 percent of the wave height remaining at 60 feet as a limit, was taken from Figure 7 which shows that 96 percent of the wave height remains at 60 feet with a smooth sand bottom, which is essentially what would remain if the ripple pitch-height ratio were infinite. Further analysis of Figure 9 reveals that if the bottom were infinitely rough, about 70 percent of the wave height would remain at 60 feet, but since no data were taken with a ripple pitch-height ratio less than one, the behavior of the curve below this point could be different than shown.

38. Percolation Tests. The results of the tests for wave energy losses due to percolation in a permeable sand bottom are shown in Table 4. All observed values are adjusted to eliminate the effect of side friction. Values were obtained for a control side of a smooth board bottom, and also a sand veneered board bottom which, it was thought, would more nearly approach the actual roughness of the sand bottom on the test side. As may be seen from Table 4, little difference was observed between the two. The results for only two sizes of sand are shown in the table because no loss was observed in the length of the tank due to percolation in tests with the 0.52-millimeter median diameter sand. Since the permeability of the 0.216-millimeter median diameter sand is even less than that of the 0.52-millimeter sand, it was assumed that no loss would be observed and no tests were run. Therefore, the results shown deal only with the sands with median diameters of 3.82 and 1.94 millimeters.

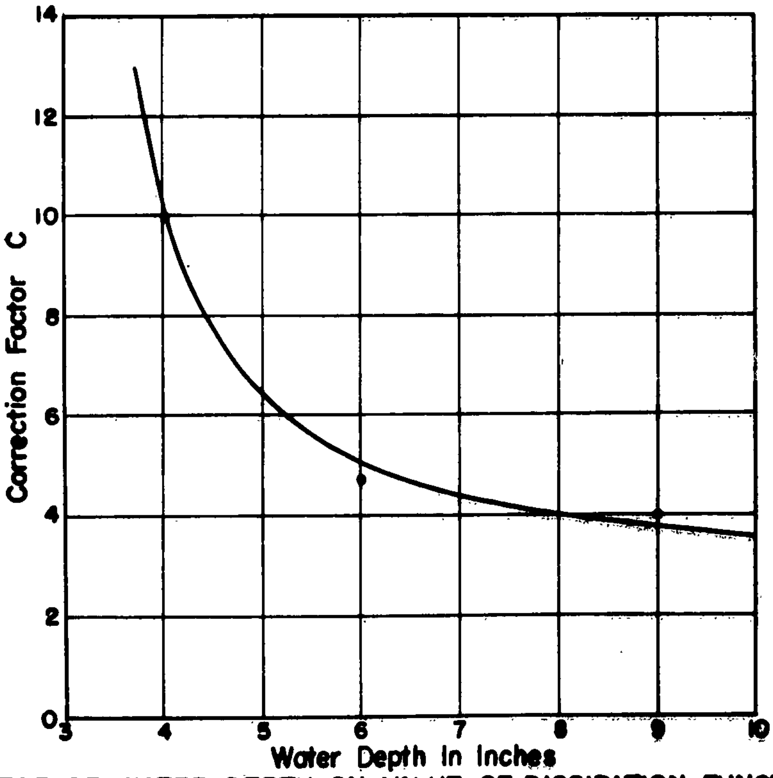
TABLE 4
WAVE HEIGHT AND ENERGY LOSSES
PERCOLATION TESTS

Note: All values of H_r are for a point 60 feet from the beginning of the beach, adjusted to eliminate the effects of side friction. H_{ros} indicate observed heights adjusted on the basis of side friction effects observed with the smooth board bottom. H_{rov} indicates observed heights adjusted on the basis of side friction effects observed with the sand veneered bottom. H_{rt} indicates theoretical height.

Run No.	Period (secs.)	Water Depth (inches)	H_I (inches)	H_{ros} (inches)	H_{rov} (inches)	H_{rt} Modified (inches)	$\frac{H_{ro}-H_{rt}}{H_{ro}} \times 100$ (%)	$\frac{H_{ro}}{H_I} \times 100$ (%)	Energy Loss (%)
					GM_e = 3.82 mm.				
1	1.27	9	3.68	1.78	1.84	1.98	-8	50	75
1	1.27	9	2.13	1.10	1.19	1.16	3	56	69
3	1.27	9	2.70	1.48	1.55	1.48	5	57	68
4	1.27	9	1.00	0.52	0.56	0.54	4	56	67
5	1.00	9	3.50	1.42	1.65	1.68	-2	47	78
6	1.00	9	2.50	1.15	1.30	1.22	6	52	73
7	1.00	9	1.20	0.50	0.58	0.58	0	48	77
8	1.00	9	1.15	0.48	0.56	0.56	0	49	76
9	0.80	9	2.70	1.54	1.57	1.38	12	58	66
10	0.80	9	1.81	1.03	1.05	0.92	12	58	66
11	0.80	9	1.20	0.60	0.65	0.62	5	54	71
12	1.27	9	3.50	1.78	1.90	1.89	1	54	70
13	1.27	9	2.20	1.22	1.31	1.18	10	60	65
14	1.27	9	1.03	0.49	0.52	0.55	-6	50	74
15	1.00	9	3.00	1.37	1.49	1.42	5	50	75
16	1.00	9	2.50	1.06	1.21	1.22	-1	48	78
17	1.00	9	1.05	0.46	0.53	0.51	4	50	74
18	0.80	9	3.50	2.14	2.16	1.74	19	61	61
19	0.80	9	2.12	1.16	1.18	1.07	8	56	69
20	0.80	9	1.10	0.62	0.63	0.58	9	57	67
21	1.27	6	2.50	0.78	0.91	0.90	1	36	87
22	1.27	6	1.00	0.42	0.50	0.34	32	50	75
23	1.27	6	2.40	0.72	0.87	0.61	30	36	87
24	1.00	6	1.00	0.37	0.44	0.25	43	44	81
25	0.80	6	2.30	0.61	0.69	0.51	26	30	91
26	0.80	4	1.10	0.34	0.43	0.15	65	39	85
27	1.00	4	1.30	----	0.33	0.16	51	25	94
28	1.27	4	1.50	0.31	0.35	0.10	71	23	95
29	1.27	4	1.20	0.37	0.40	0.10	75	33	89
					GM_e = 1.94 mm.				
72	1.27	9	2.06	1.48		1.44	3	72	48
73	1.27	9	1.15	0.82		0.80	2	71	49
74	1.27	9	3.00	2.26		2.11	7	75	43
75	1.00	9	1.98	1.36		1.40	-3	69	53
76	1.00	9	1.25	0.89		0.89	0	71	49
77	0.80	9	1.30	0.96		0.95	1	74	45
78	0.80	9	1.68	1.16		1.21	-4	69	52



EFFECT OF RIPPLE PITCH-HEIGHT RATIO ON FRICTION LOSSES
Fig. 9



EFFECT OF WATER DEPTH ON VALUE OF DISSIPATION FUNCTION
PERCOLATION TESTS
Fig. 10

39. The theoretical wave height reamining at 60 feet shown in Table 4 was computed using the equations as given by Putnam, but in this case the equations were somewhat modified. The first calculations attempted using the theoretical equations gave energy losses so large that the waves theoretically disappeared before they had reached a distance of 60 feet from the beginning of the beach. This discrepancy was overcome by dividing D_p, the dissipation function, by a constant, (C) thus reducing the theoretical dissipation and the magnitude of the height loss. All the theoretical results shown in Table 4 were computed by dividing D_p by 4, and are termed modified theoretical values. Comparison of the modified theoretical and observed heights remaining at 60 feet (smooth side with gravel vs. modified theoretical) reveals that the modified theoretical results agree very well with the observed results for both material sizes and a water depth of 9 inches. For other water depths and the 3.82 millimeter sand, even the modified theory breaks down and further modification is necessary. It was found that a different correction factor is required for each depth. The factors determined are shown in Figure 10 plotted versus the water depth. From the graph, it appears that as the water depth approaches zero, C approaches infinity and the theory breaks down. This is logical because gravity waves of the magnitude of those used in these tests are not too stable in 4 inches of water and would be completely unstable in depths approaching zero. As the water depth increases, C appears to approach a minimum value as a limit, but due to the narrow range of depths tested, this limit cannot be determined with the data available.

40. In comparing the computed and observed heights over the 3.82-millimeter sand, it was felt that the results observed with the coarse sand veneer on the impermeable side were more nearly comparable since the friction effect of the roughness of the bottom is thereby excluded from the computation of percolation losses. Later computations of the friction effect of the roughness of the bottom indicated that this factor accounted for a loss of approximately 4 percent of the initial height, which compares rather favorably with the 4.2 percent of the initial height observed in the continuous friction test. Since the friction effect of the smooth sand bottom was relatively small, the sand on the smooth side was omitted in the runs with the 1.94-millimeter sand.

CONCLUSIONS

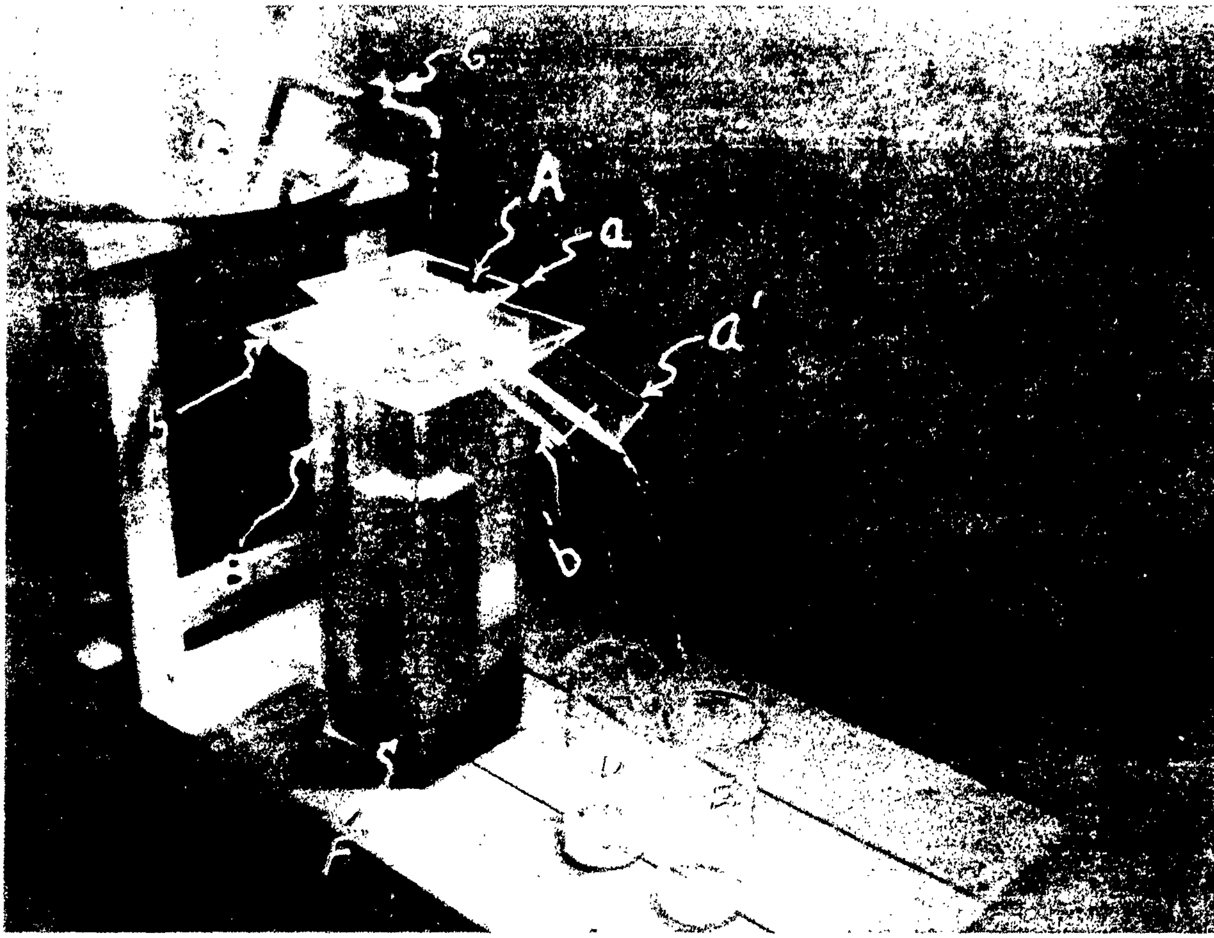
41. Friction Tests - Natural Ripples. The limited data obtained from one run, which indicate that the loss of energy is dependent upon the stage of ripple formation for the particular wave period and height, and that the energy loss increases to a maximum rate during formation of the ripples, thence decreases nearly to the initial value after completion of the ripple formation, are substantiated by data from a second run in which the wave period was varied and the ripples formed by the preceding wave train were utilized as test ripples. The maximum rate of frictional loss agrees reasonably with the rate computed by the theoretical method of Putnam and Johnson. The maximum frictional loss of energy observed in the first run amounted to about 26 percent, and the average friction loss in the second run was 14 percent of the initial energy.

42. Friction Tests - Artificial Ripples. These tests indicate that the theoretical equations are unsatisfactory for determination of bottom frictional losses under the conditions tested for artificial ripples dissimilar to natural ripples. The maximum energy loss under these conditions approached 60 percent. Tests with artificial ripples more similar to the natural ripples resulted in reasonable agreement of test and theoretical values of loss of wave height. The maximum loss amounted to 23 percent of the wave height with a corresponding energy loss of about 41 percent. The tests indicate that friction losses increase with decrease of ratio of ripple pitch to ripple height.

43. Percolation Tests. These tests indicate that the energy losses by percolation are far less than theoretical values, it being found necessary to divide the dissipation function by a correction factor of from 4 to 10 to obtain reasonable agreement. The correction factor appears to be an inverse function of water depth. One test resulted in the general conclusion that energy losses by percolation are insignificant for sands of low permeability. The tests also show that in model tests where sands larger than 0.5 millimeter are used as a beach, percolation losses can be very large, especially with sand sizes larger than 2 millimeters.

44. Guide for Future Studies. The tests indicate that if further model studies are conducted for friction and percolation effects on wave energy losses, the wave flumes in which the tests are conducted should be deep enough to test greater depths of water with larger wave heights and periods, and wide enough to render the effects of side friction negligible.

45. The tests also indicate that the present method for determining K_f, the friction factor for the interaction between waves and sand bottoms, is inadequate and needs to be further investigated. Further investigation of K_f would necessarily include some quantitative investigation of the relationships between sand size, water depth, wave characteristics, and ripple characteristics, especially where R/p is less than 1.



PERMEAMETER USED IN PERMEABILITY TESTS
Fig. A-1

APPENDIX

1. Permeameter. The sands used in the tests for wave energy losses by percolation were tested for permeability in the constant-head permeameter constructed of 1/4-inch plexiglass which is shown in Figure A-1. The center container A holds the sand to be tested and has a screen on the bottom to retain sand while water flows through. Part B is a waterproof container. Both parts, A and B, are equipped with troughs (a and b) which fit around the outside near the top and are drained by a small overflow troughs a' and b'.

2. In order to determine the permeability of a sand, it is placed in Part A, Figure A-1, which is then placed inside part B, resting on an open metal stand, F. The hydraulic head (Δ h), being the vertical difference between the water surfaces in containers A and B, is then adjusted to any desired value by raising or lowering the metal stand F. Next, a 50-gallon reservoir, C, is filled with water and allowed to stand until the water temperature has risen to essentially room temperature so that all excess gases in solution will escape. By opening valve G, the water is allowed to run into part B until it is filled. This is done to allow the water to rise in the sand and reduce the possibility of trapping air bubbles in the pores of the sand. After part B is filled, the flow is directed into the top of part A in such volume that part of it runs over and is carried off by troughs a and a' to be caught as waste in container E. The portion of the flow directed into part A that does not run over (Q) percolates down through the sand, out the bottom of part A, and flows over the top of part B through troughs b and b' into container D, where it is caught and measured. A record is kept of the following: water temperatures, length (L_s) of sand column in container A, area (A) of sand column in container A, time in which a certain quantity of water runs into container D after the flow has reached a steady state, and the difference in hydraulic head (Δh). Using these observed quantities in Darcy's equation,

$$Q = \frac{k A \Delta h}{L_s}$$

the transmission coefficient (k) for the sand is computed. Several runs are made keeping all variables constant and measuring the discharge (Q). The difference in hydraulic head (Δ h) is then changed and several more runs are made. The transmission coefficient (k) is computed for each run and an average value of k is used to compute the coefficient of permeability (K) in feet^2 using the equation:

$$K = \frac{k\mu}{\rho}$$

Care is taken at all times to keep the flow through the sand laminar, the criterion being that departure from laminar flow begins at Reynolds numbers between 1 and 10(7). These Reynold's numbers were computed from:

$$R_e = \frac{(\rho)\ (V)\ (GM_e)\ (3.28 \times 10^{-3})}{\mu}$$

where the symbols and their dimensions are as defined on page i.

3. The permeabilities of the sand to be used in the friction and percolation tests were determined by using the apparatus described in the preceding paragraph. As a check, the permeabilities were also computed using the equations given by Krumbein and Monk(8) for sand permeability as a function of the phi standard deviation and the median diameter:

$$K = 760\ (GM_e)^2\ e^{-1.31\,\sigma\phi}$$

where K is in darcys. The measured and computed values of permeability for the test sands are in good agreement, as shown in the following table.

VALUES OF PERMEABILITY COEFFICIENT (K)

Sand Size (Med. Dia. in mm)	Phi St'd. Deviation	k (cm./sec.)	$K \times 10^{10}$ Computed ($ft.^2$)	$K \times 10^{10}$ Measured ($ft.^2$)
3.82	0.71	4.93	466.	483.
1.94	0.13	2.00	256.	231.
0.52	1.05	0.057	5.52	6.02
0.216	0.42	0.0215	2.16	2.25

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